

In 1991, I published an original paper in Japanese describing a musical theory that integrates rhythm, scale, and tone using the self-similitude of the Golden Mean [3]. From this viewpoint, any existing musical forms are too finite.

When I participated in the international Bamboo Congress 1995 in Bali, I watched an Indonesian polyrhythm performance. This experience allowed me to compose a new piece making use of my musical theory. This is the Fibonacci Kecak [4].

The Real Kecak System is a general extension of the Fibonacci Kecak. It is interactive computer music. First, I wrote the program in N88Basic in 1999; then, I rewrote it in MAX for Macintosh in 2000 [5].

3. Mathematical Principle

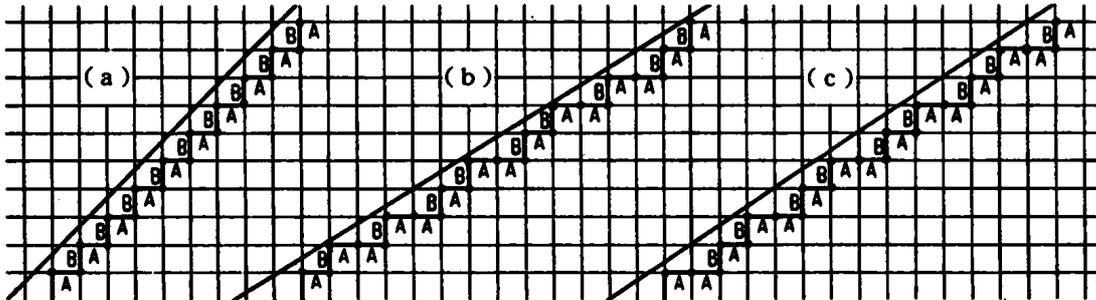


Figure 2: General Extension of the Fibonacci Sequence [6]

3.1. Extension of the Fibonacci Sequence. It was well known that the Fibonacci Sequence was generated by the line whose tangent was the golden mean on the 2D coordinate lattice as shown in Fig. 2(b).

In 1992, I published that such a sequence of letters could be extended to any real number as shown on Fig.2 [6]. The example (a) is 1/1, the example (c) is 2/3. Generally any rational number generates the periodic sequence of letters. The period is the sum of the numerator and the denominator of the fraction, e.g. in case of Fig.2(c); the sequence of letters repeats every 5 letters.

If the tangent is an irrational number, the sequence will never repeat.

I suggest that such a general sequence of letters should be called a “Real Sequence”.

3.2. Substitution and Convergence. In the mean time, it is well known that there is another method to get the Fibonacci Sequence with the following algorithm.

Initial sequence is “AB”.

Transform “A” to “AB” and “B” to “A”.

Then we get 2nd sequence “ABA”. Transform again by the same way.

Then we get 3rd sequence “ABAAB”. Go ahead.

4th sequence is “ABAABABA”.

And so on...

I realized that the above algorithm corresponded to the hierarchy of convergences derived from the continued fraction of the Golden Mean 0.6180339....

Initial sequence	AB	1/1	$0 + \frac{1}{\quad}$
2 nd sequence	ABA	1/2	$\frac{1}{1 + \frac{1}{\quad}}$
3 rd sequence	ABAAB	2/3	$\frac{1}{1 + \frac{1}{1 + \frac{1}{\quad}}}$
4 th sequence	ABAABABA	3/5	$\frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\quad}}}}$
5 th sequence	ABAABABAABAAB	5/8	$\frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\quad}}}}}$
6 th sequence	ABAABABAABAABABAABAABABA	8/13	$\frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\quad}}}}}}$
and so on...			...

The numeral of the denominator is number of letters “A”.

The numeral of the numerator is number of letters “B”. Everything is the Fibonacci number.

Self-similitude of the Golden Mean is represented on the above schema exhaustively. I substituted sound for letters of each sequence. So to speak the music was rhythmic counterpoint. We can enjoy the phantasmagoric moiré of rhythm patterns. I believe that there had not been such quite successful music using the Golden Mean.

This is the principle of the Fibonacci Kecak (1995).

3.3. Extension of the Fibonacci Kecak. On the other hand, any real number has their own hierarchy of the convergences derived from a continued fraction. Each convergence has their own periodic sequence derived from the Real Sequence as I defined above. Therefore the Fibonacci Kecak can also be extended to any real number. The Real Kecak System plays polyrhythm music consists of these sequences of convergences. We can exhaustively make the best use of the self-similitude of irrational numbers for music. Our ears can recognize the characters of real numbers.

Since the number of real numbers is infinite, there are endless kinds of musical forms. Since most of the real numbers are irrational numbers, most of the music made should be non-periodic and eternal.

This is the principle of the Real Kecak System. We can substitute anything into “A” and “B”.

4. Manual of the Real Kecak System

You can input any real number. The Real Kecak System will display its expansion into continued fractions and convergences based on the Euclidean Algorithm [7]. Each n-th convergence corresponds to a planet in the Real Kecak System. Each planet plays their unique rhythm. The circumference of the orbit represents the period of the rhythm pattern.

When you input an integer or rational number, you hear a simple periodic rhythm. It is so simple that you should be tired of it soon. Such a system has very few planets.

On the other hand, when you input an irrational number, e.g., $\sqrt{2}$, $\sqrt{3}$, or e , the music is fantastic, and we do not get tired of it. The musical texture goes on changing little by little beyond the astronomical period. We enjoy it as ethnic music.

5. Fibonacci Kecak

If the Golden Mean (1.6180339... or 0.618034...) is entered, the Real Kecak System plays the Fibonacci Kecak. The system has the most number of planets, because approximating the Golden Mean using fractions is more difficult than approximating any other irrational number. In other words, in the case of the Fibonacci Kecak System, there is rarely conjunction.

What follows is the calculation of the length of time during which the Fibonacci Kecak can keep changing. We can get the solution by calculation the L.C.M. of the following fibonacci numbers.

2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597 2584 4181 6765

If each beat is 150 milliseconds long, the music piece will not be repeated for 522,629,966,200,000,000 years.

Most astronomers consider that our universe is 15 billion years old. We need 35 million times the life of our universe to listen to an entire piece of music. This length of time is not easy for humans to grasp. But we should not say that it is not realistic.

On the other hand, if a single-digit integer is entered, the music is repeated in just a few seconds. Most people find the Fibonacci Kecak very exciting, i.e., the Real Kecak System evidences that the Golden Mean is the critical real number. Although it is the simplest structure, it generates the most diverse gestalt. That is why Nature very often uses the Golden Mean. Therefore, I also use the Golden Mean actively for artistic purposes.

So to speak, the Real Kecak System is a typical fruit created by Pythagoras being equipped with an Ouroboros Engine.

6. Installation with lights

In 2001, I created an installation called “Stern-warte” [8] in Tokyo as shown in Figure 3. I built a maze based on the Penrose Tiling all over the floor. People could walk between the pentagonal mounds of sand as if they were ants. This work was an experimental city design. I installed lamps over each pentagonal mound. They twinkled diversely being synchronized with the sound of the Fibonacci Kecak. A computer controlled both sound and light.

The architectural form is completely homologous with the musical form. Architecture and music were elegantly integrated in a common space and time. I believe that it was a completely novel experience for the visitors. The installation is not there now, but I would like to build such a space as a permanent sculpture in the near future.

7. Concert and Workshop

The Real Kecak System can be played as instrumental music. I established a percussion ensemble called “ENSEMBLE STARCAGE” with my friends. Sometimes, I held workshops to instruct children on the Real Kecak using bamboo sticks. They enjoyed it very much.

Furthermore, I try to use this system to help schizophrenic patients during their psychoanalytical treatment [9].

Currently, some Japanese professional musicians are using my musical theory: Mr. Chiharu Wakabayashi, Ms. Tamami Tono, Mr. Takeshi Isogai (Ubartmar), and Ms. Naoko Maeda. Once I rewrote a score on paper, then I made several concerts with Tamami Tono who plays the SHO which is a Japanese traditional bamboo flute.



Figure 3: Installation “Stern-warte” Photo: T Tsukagoshi

Acknowledgement

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References

- [1] Akio HIZUME, *Life and Architecture*, 1987. Graduation thesis of Kyoto Technical and Textile University.
- [2] Akio HIZUME, *STAR CAGE: New Dimension of the Penrose Lattice*, *FORMA*, 9, 259-272. 1994.
You can see my artistic activities in website. (<http://homepage1.nifty.com/starcage/>)
- [3] I published the idea as a pre-print paper titled “Devil’s Music” in 1991.
Next, I rewrote it as a couple of applications to the Japan Patent Office in 1992;
Publication number: 05-323958 and Publication number: 06-308953. (You can read all English abstracts of publications translated by the Japan Patent Office in their website.)
Then I published a scientific paper in Japanese;
Akio HIZUME, *The Form of the Golden Music*, *Culture on Form Bulletin II*, KO-SAKU-SHA. 1994.
- [4] *FIBONACCI KECAK*, presented at *ISIS-SYMMETRY*, Washington D.C., Aug. 1995.
- [5] Akio HIZUME, *REAL KECAK SYSTEM* (paper in Japanese), *MANIFOLD #02*, 2001.
- [6] Figure 2 is reprint from Akio HIZUME 1992,1994 [3].
- [7] I know that any computer is practically finite. I programmed that his calculation would be stopped when the n -th convergence’s errors in the range became less than 0.03 percent.
- [8] This exhibition was held as collaboration with Tomoko NINOMIYA at gallery MUKURI Daikanyama, Tokyo, Japan, Aug. 2001.
- [9] In 1999, I began to collaborate with Dr. Kazushige SHINGU who is Japanese psychoanalyst.