Poly-Twistor

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1. Pentagonal Gravity and Hexa-Twistor

In 1997, I published a series of works on Star Cages which consist of three dimensional curves, not straight lines, and are based on the MU-MAGARI [1]. There are many kinds of stars. Please look at Figure 1 to see some examples from the series. I named the series "Pentagonal Gravity" because the curve reminds me of a ray of light bending next to a star.

All of the Pentagonal Gravity series consists of 30 curved lines which are all the exact same shape. No curved line shares a common plane. Made of spineless wires, the construction becomes very stable when the star shape is achieved.

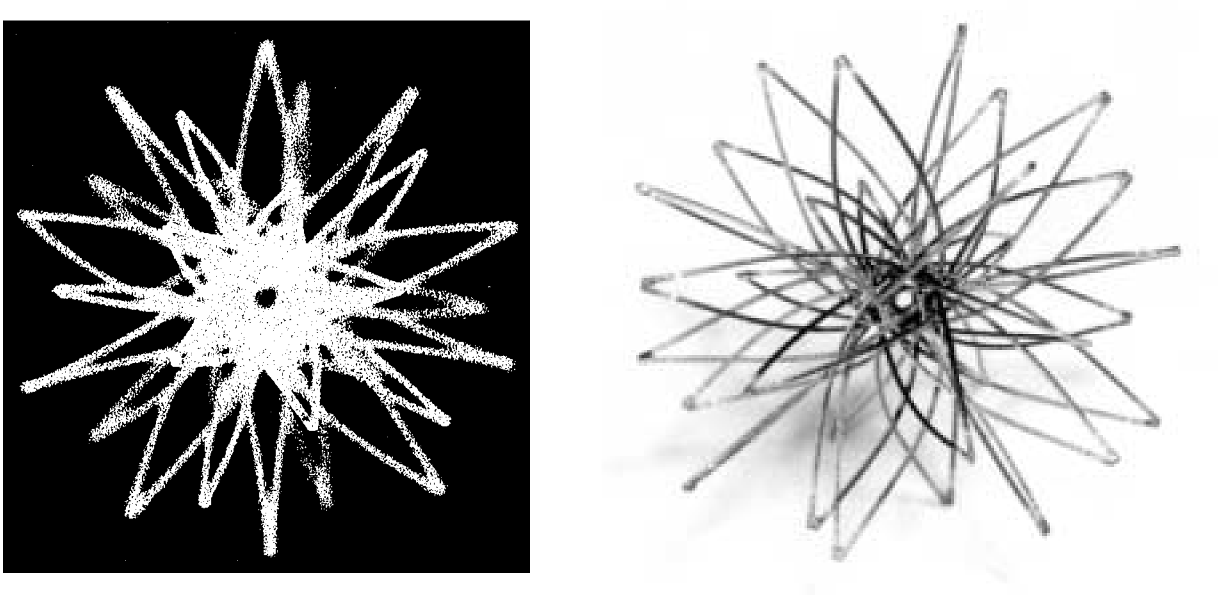


Figure 1: *Some of the Pentagonal Gravity*

I published another series at the same time called the "Hexa-Twistor (1997)" [2]. The external form of the Hexa-Twistor is spherical, but it is not a simple structure. Like the MUMAGARI, both the inner and outer layer crusts are woven and continue each other. In 3D space where the curvature is zero, the straight line will never come back to the starting point, however, in the geometric space represented by the Hexa-Twistor, the straight line will come back.

In another way, this can be viewed as a study of curved space which has identified each opposite surface of a dodecahedron. There are various ways to identify the opposite sides of a dodecahedron [3].

The Hexa-Twistor consists of 6 closed 3D curves. Each curve coils around the surface of a torus. It is understandable that the Hexa-Twistor consists of six “helical-tori” which will be mentioned later.

I actually made two models using cable in 1997. These two are defined as ***[5/2]*** and ***[5/1]*** with a classification around the "helical-torus". I drew the following Figures 2,3 using a computer in 1999 to cover all forms of the Hexa-Twistor [4].

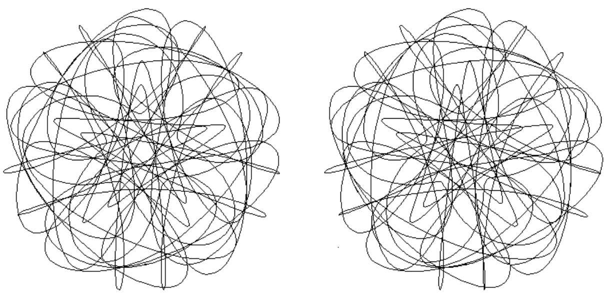


Figure 2: *Hexa-Twistor **[5/2] (stereo: cross your eyes.)***

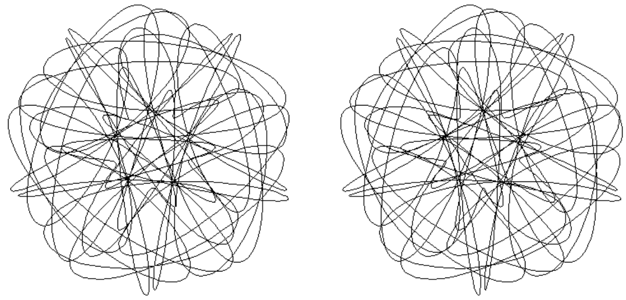


Figure 3: *Hexa-Twistor **[5/1]***

2. Topology of the helical-torus

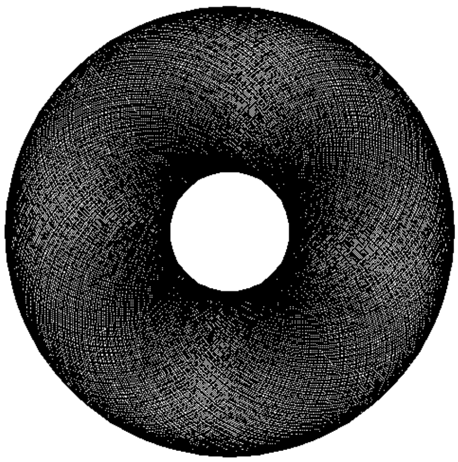


Figure 4: *Helical Torus **[1.6180339⋯]***

What follows is the definition of the helical-torus [5]. First, draw any straight line on the 2D coordinate lattice. This line crosses the square lattices successively.

Second, map each crossed square to one square. Let's call it the "unit reference square", which we regard on opposite sides of the same edge. As is generally known, such a square is equivalent of the torus surface topologically. Thus any helical-torus is defined by the gradient of a straight line.

Fig. 4 is an example where the gradient is the golden mean. In this case, the line sweeps the surface of the torus endlessly because the golden mean is an irrational number. The golden mean defines the most efficient coil to string around the torus.

If a two-dimensional creature who lives on the torus surface decides to start an around-the-world trip, I would suggest to him to leave in the golden mean direction. He needs to know the curvature of his universe in advance. After departure he may be able to see his whole universe for a short period of time. However, he could never arrive back home as long as he keeps advancing.

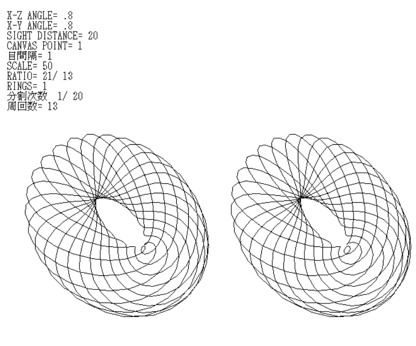
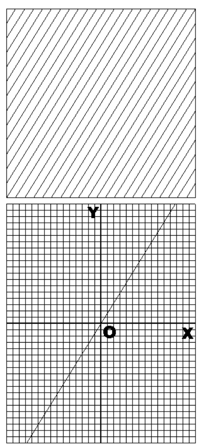


Figure 5: *Helical Torus **[21/13]***



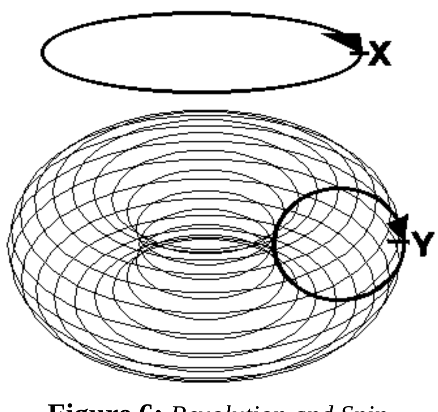


Figure 6: *Revolution and Spin*

Fig. 5 is a rational helical-torus whose gradient is 21/13, which is a convergence of the golden mean. The traveler can come back home in this way because the ratio is a rational number. In a manner of speaking, this journey is a package tour of the torus world.

Generally, in the case that the gradient is an irrational number, the surface of the the torus should be filled with helical lines. n the case of a rational number, the sweeping should be stopped and there will be periodic oscillation.

The helical torus can be classified topologically as follows. There is no consideration about size or ratio.

The helical-torus can be classified by two parameters, that is, ***number of strings**** and ***[frequency]***.
The x coordinate on the reference square corresponds to the revolution point around the open hole.
The y coordinate on the reference square corresponds to the spin around the closed tube.

The ***[frequency]*** of the helical torus is the frequency of spins per a revolution (Fig. 6). The frequency is the equivalent tangent (gradient) of the line on the surface. The ***[frequency]*** can be any real number and makes a one-to-one correspondence with the form of the helical-torus. The sign of the ***[±frequency]*** decides the chirality of the helical-torus.

The ***number of strings**** must be natural number.

If the ***[frequency]*** is not an integer, the helical-torus makes a knot.

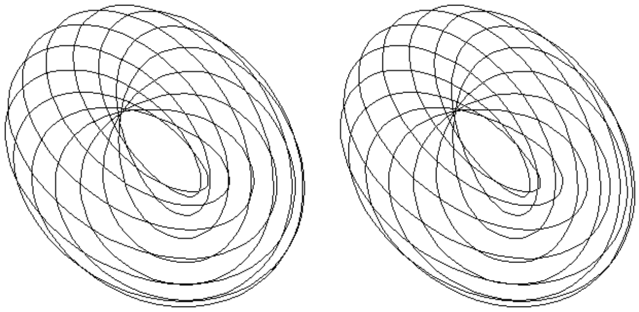


Figure 7: *Helical-Torus **[8/13]***

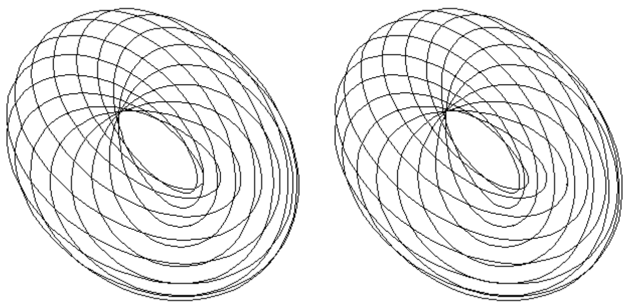


Figure 8: *Helical-Torus **3*[3/5]***

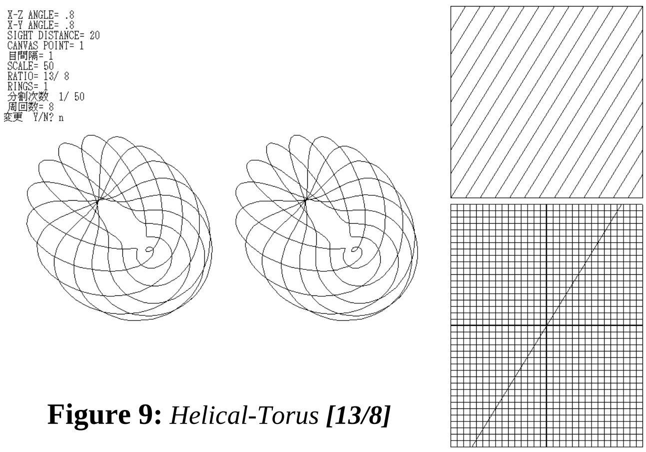


Figure 9: *Helical-Torus **[13/8]***

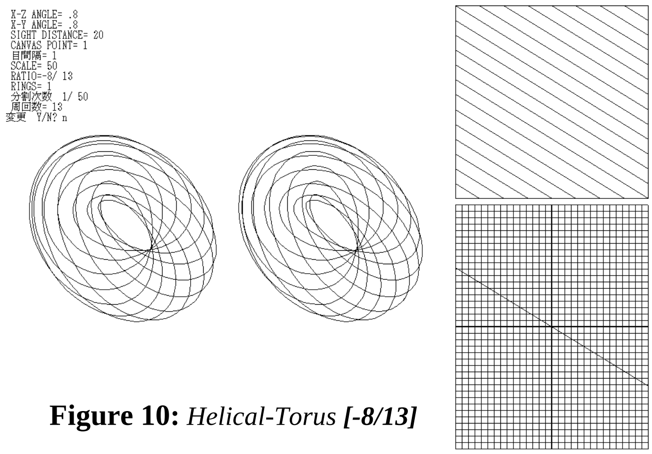


Figure 10: *Helical-Torus **[-8/13]***

If the sequence of one helical torus is ***[q/p]*** and another is ***[p/q]***, the relationship is called a "duality", e.g., see the relationship between Fig. 7 and Fig. 9. Of special note are ***[1/1]*** and ***[-1/1]*** which are self-dual.

If the product of the frequency of two helical-tori is minus one, they are orthogonal to each other, e.g. see the relationship between Fig. 9 and Fig. 10.

3. Genaral Form of the Hexa-Twistor

The general Hexa-Twistor consists of six helical tori whose frequencies are all the same. The equator planes of any two tori cross each other at an angle of arc tangent 2. It is the face angle of a dodecahedron. Note there is another chirality what the two equator planes cross each other. For example, however both Fig. 11 and 12 consist of the same helical tori, the structures are quite different each other.

Therefore there are four stereoisomers in the Hexa-Twistor. [6]

In the Hexa-Twistor, the amplitude of the helix is an additional parameter. We can choose a suitable amplitude where the helical-tori avoid each other.

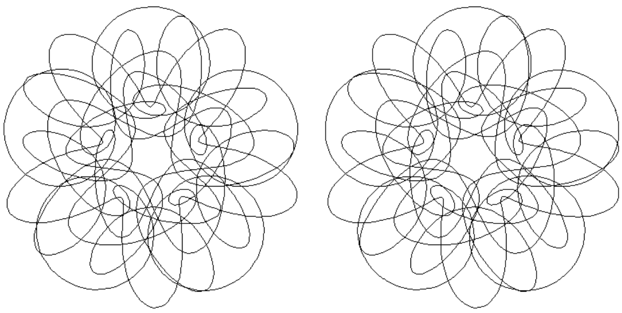


Figure 11: *Hexa-Twistor **[5/1]***

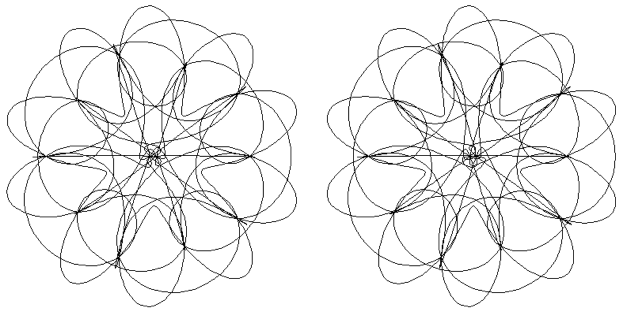


Figure 12: *Hexa-Twistor **-[5/1]***

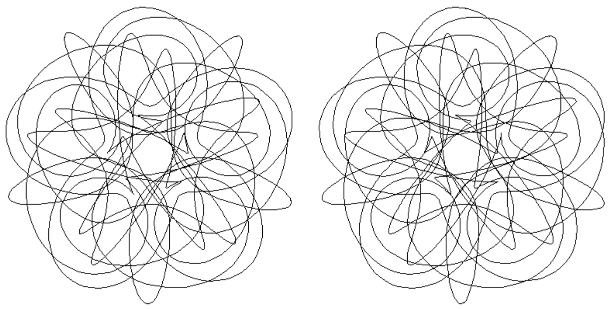


Figure 13: *Hexa-Twistor **-[5/2]***

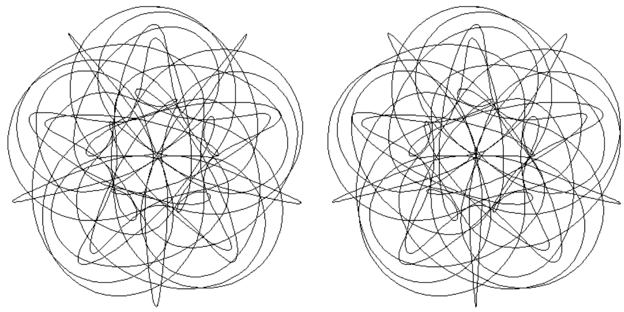


Figure 14: *Hexa-Twistor **-[5/3]***

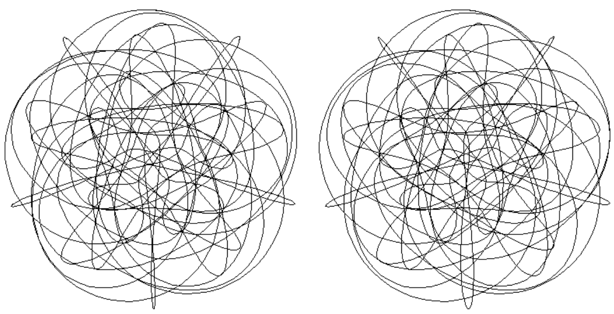


Figure 15: *Hexa-Twistor **-[5/4]***

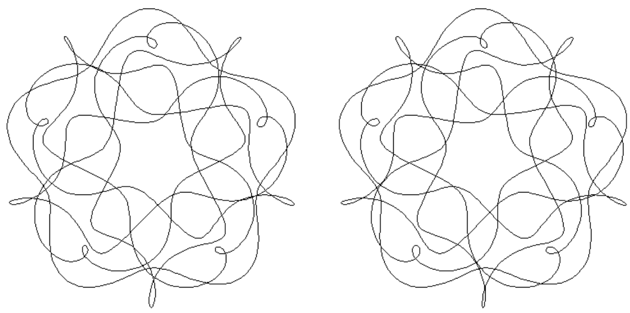


Figure 16: *Hexa-Twistor **-[5/1]0.15***

4. Poly-Twistor

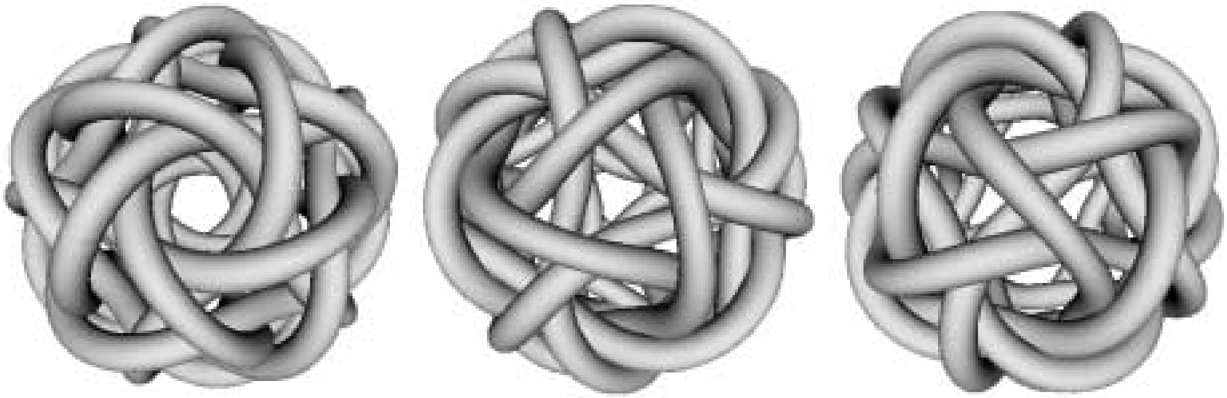
In 2002, I evolved the Poly-Twistor which is based on the symmetry of the Plato's Solids[1]. There are four kinds of Poly-Twistors. The essential facial angles are as follows:

Deca-Twistor: $\mathrm{Sin}^{-1}(2/3)$
Hexa- Twistor: $\mathrm{Tan}^{-1}2$
Tetra- Twistor: $\mathrm{Cos}^{-1}(1/3)$
Tri- Twistor: $\pi/2$

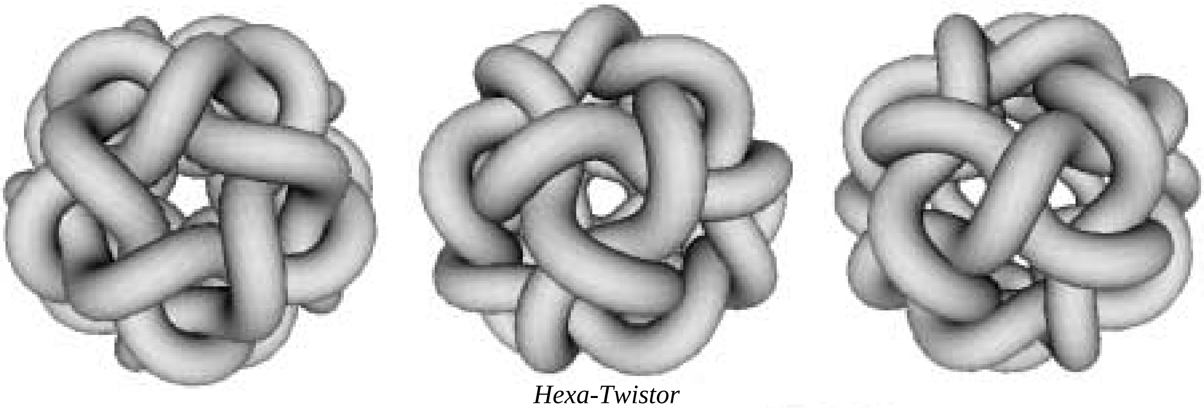
Later I may create a Poly-Twistor program which consists of 15 helical-tori.
Fig. 18 is the simplest one of each class.

I used VRML and Javascript to make the program interactive. You can change any .parameters of the frequency of the helical torus, the thickness of the helix, or the amplitude of the helix. There is a huge diversity of Poly-Twistors. Fig. 19 shows some of them.

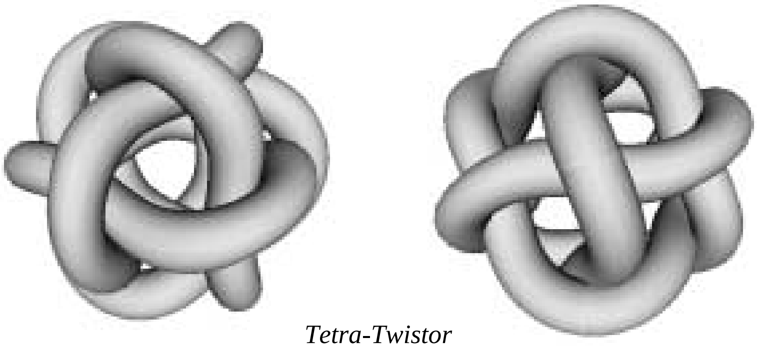
Figure 17: *Deca-Twistor*



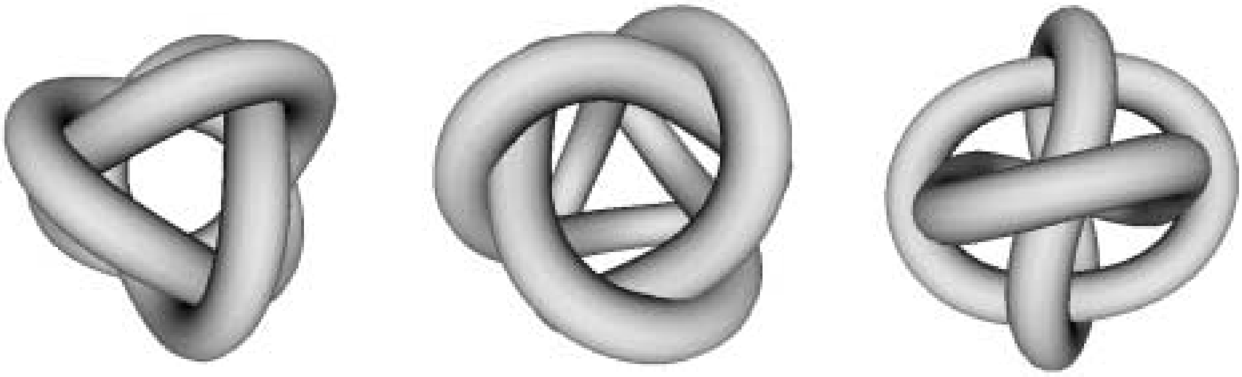
Deca-Twistor



Hexa-Twistor

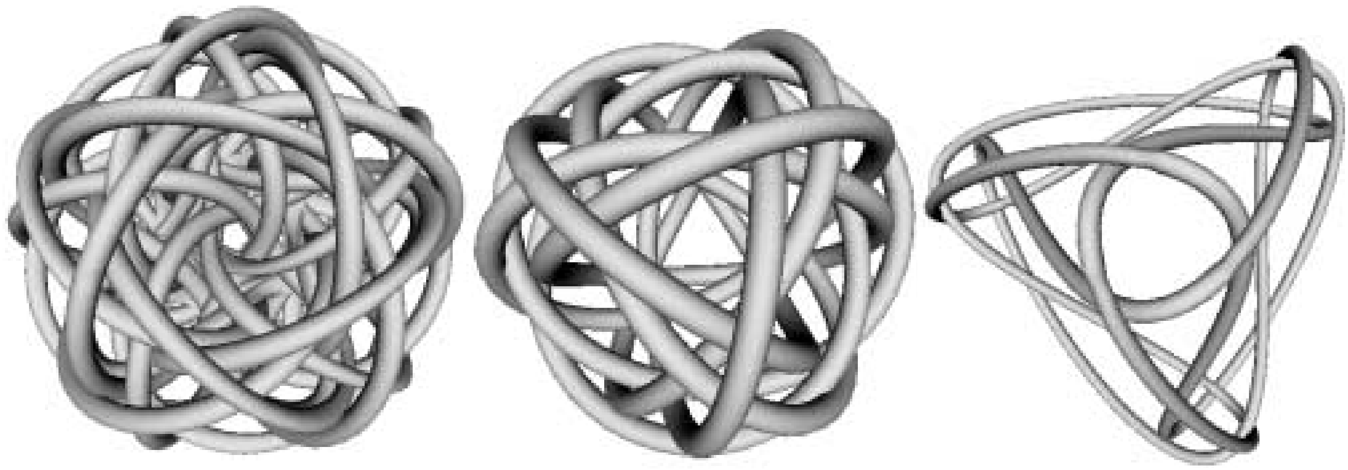


Tetra-Twistor

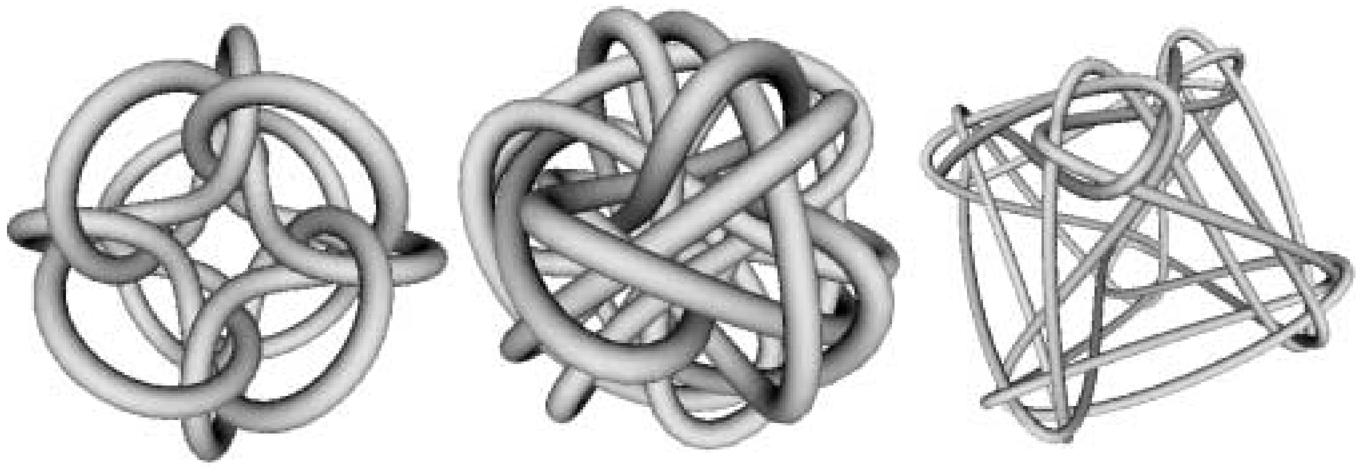


Tri-Twistor

Figure 18: *Poly-Twistor*



DECA [3/2] *TETRA -[3/2]* *TRI [3/1]*



*TETRA -**[3/1]*** *TETRA -**[3/2]*** *TETRA **[3/1]***

Figure 19: *Diverse Poly-Twistors*

References

[1] The MUMAGARI is a 3D quasi-periodic chiral lattice consists of 6 axes.
[2] I presented it to Prof. Koji MIYAZAKI and Prof. Roger PENROSE. Prof. Penrose suggested me the name of the structures when we met personally in 1997 in Kyoto.
[3] W.P.Thurston did advanced study about the subject. (Thurston 1984)
[4] Akio HIZUME, *Hexa-Twistor*, 2000, MANIFOLD #01.
[5] I got this idea when I found a way to generated the Fibonacci Sequence.
Akio HIZUME, *Life and Architecture*, 1987. Graduation thesis of Kyoto Technical and Textile University.
[6] The Hexa-Twistor and Pleiades also have four stereoisomers.

Acknowledgement

I would like to thank Junichi Yananose who helped me to develop VRML image.
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