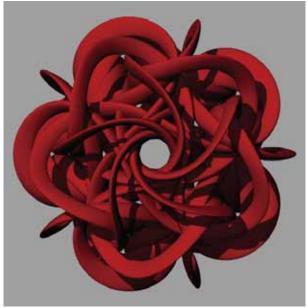
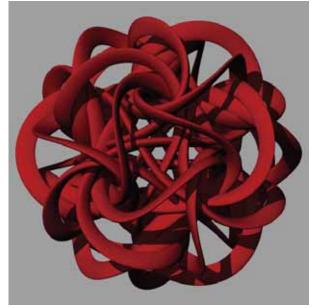
## Poly-Twistor of Precessed Elliptical Orbit

February 2025 (The 1st edition)
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5 fold symmetry

3 fold symmetry

Fig. 1 Poly-Twistor of Precessed Ellipse Torus (2025)

Fig. 1 above shows the latest work (2025) in a series of what I call Poly-Twistor, which I began studying in 1997.

The center of a black hole or star is a super-dense singularity, curving space to a negative curvature, and the surrounding bodies take elliptical orbits with the singularity as their focal point. This is the law of gravity as revealed by Kepler, Newton and Einstein. It is also known that the entire system is in more or less precessional motion around the focus of the elliptical orbit. If observed from outside the system, the trajectories of the surrounding bodies would appear to be a sweeping motion on the torus surface.

The new work uses several of these 'precessed elliptical orbits' and is constructed on the basis of the symmetry of the Platonic solid. It is also reminiscent of the orbits of a group of electrons caged around an atomic nucleus. I have produced and presented this type of structure in a series entitled 'A Form of the Cosmos, A Form of the Atom'. [1]

### 1. 1. Helical Torus

The early Poly-Twistor consisted of a basic form called the 'Helical Torus'. Let's start by confirming that a torus is defined by a major circle, a minor circle, longitude and meridian. (Figure 2)

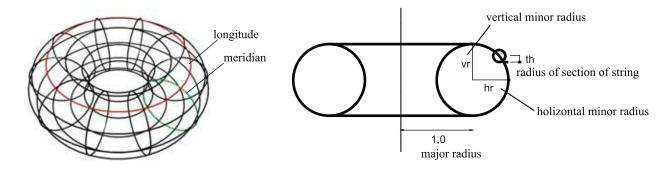
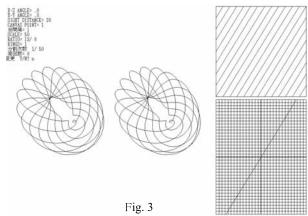


Fig. 2 Components of the torus

In the left diagram, the red circle is the longitude and the green circle is the meridian. In the diagram on the right, the distance from the center of the torus to the center of the meridian (small circle) is the major radius. For simplicity, the major radius is set to 1. The minor circle can also be an ellipse. *th* is the radius of the orbital string. The cross-section of the string is not limited to a circle, but can also be elliptical or polygonal.

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Helical Torus with frequency 13/8 (stereo)

As shown in Fig. 3, the Helical Torus is created by the synthesis of the longitudinal rotation A and the meridian rotation B, and the ratio A/B is defined as the 'frequency'. Here, anti-clockwise is defined as plus.

In particular, when the frequency is an integer (A/1), the shape of the helical torus presents the aspect of de Broglie's 'standing wave'. (Fig. 4, leftmost 5/1) When the frequency is non-integer, the Helical Torus forms various knots. (Fig. 4)

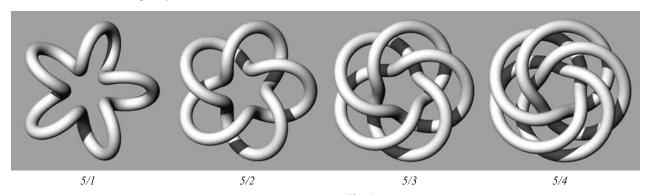


Fig. 4
Helical Torus of various frequencies

### 1. 2. Poly-Twistor as a classification of 3D torus

It is known that a 2D torus with n holes (denoted as genus n) can be topologically formed by identifying the opposite sides of a  $4 \times n$  gon. Therefore, there are infinite species of 2D torus. (Fig. 5)

As a development towards 3D torus, an attempt was made in 1997 to identify the opposite faces of a regular dodecahedron. (Fig. 6) This is a six Helical Tori structure. In the same year, when I handed the prototype model to Sir Roger Penrose during his stay in Kyoto, he named it Hexa-Twistor on the spot.

Subsequently, not only Hexa-Twistor based on the Dodecahedron, but also Cube, Octahedron, Icosahedron and Rhombic-triacontahedron were covered by the method of identifying their opposite faces and collectively named Poly-Twistor. (Fig. 7) While there are infinite types of 2D torus, there are only five types of Poly-Twistor, a representation of a 3D torus: three rings (Tri), four rings (Tetra), six rings (Hexa), ten rings (Deca) and 15 rings (XV).

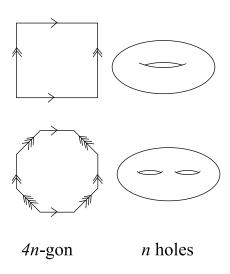


Fig. 5
There are infinite species of 2D torus



Fig. 6 Handmade Hexa-Twistor (1997) by identifying dodecahedral faces.

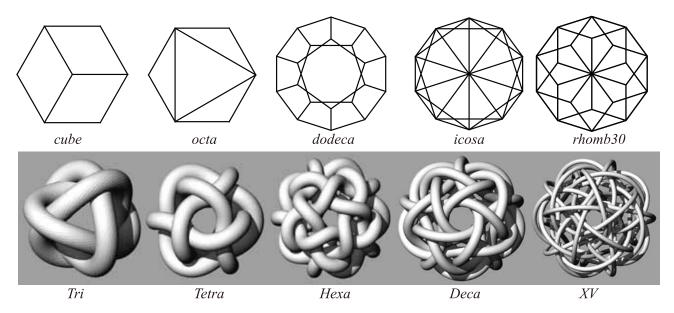


Fig. 7
Classification of the Poly-Twistor

In Tetra, Hexa and Deca, there are plus or minus chirality in the angle between the orbital planes of any two Helical Tori, so there are strictly eight types. (Fig. 8)

Furthermore, all eight types have mirror images, so more strictly speaking, they are classified into 16 types.

Poly-Twistor was a study as a topological classification of 3D tori. Initially, it was exclusively a CG-based exploration using VRML, but since 2013 it has been ported to stl data for 3D printer. The joy of having a model that I can actually touch, beyond the CG image, is indescribable. I became obsessed with prototyping topologically different models, and before I knew it, the total number of actual works had reached more than 60 (not including mirror images). The catalogue in Fig. 9 contains most of the prototyped models. [2] Fig. 10 shows some of them.

## Chirality Chirality

Fig. 8 Chirality between two Helical Tori

# Poly-Twistors Periodic Table Classification of 3D Tori Catalog of 3D Printer Models updated in Jan. 2018 ©Akio Hizume akio@starcage.org http://www.starcage.org

Fig. 9 Poly-Twistor catalogue

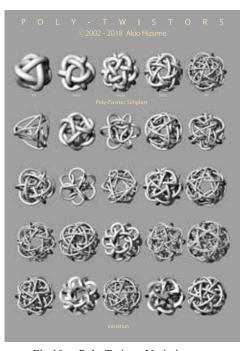


Fig.10 Poly-Twistor Variation

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### 1. 3. The form of the Cosmos

According to Einstein, "The Cosmos is finite, compact and without boundaries. Light, travelling straight ahead, will eventually regress back to where it came from." What form does that return take, is such a question Poly-Twistor's search. One can hypothesized that the real Cosmos is one of these. It is a geometrical question inspired by the non-Euclidean geometry of Riemann, Lobachevsky and others, the Poincaré conjecture and Einstein's theory of relativity.

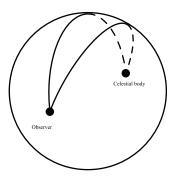


Fig. 11 Cosmos with positive curvature

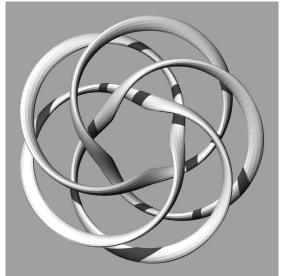
What happens on a 2D sphere also happens on a 3D sphere

There is another candidate for the form of the Cosmos other than a 3D torus, which is finite and has no boundaries. It was the 3D sphere (hypersphere). Before I became interested in the 3D torus, I thought that this was the only possible macroscopic structure of the Cosmos. Perhaps Riemann, Poincaré and Einstein thought so too. The Argentine novelist Jorge Luis Borges' 'The Library of Babel' (1941) was indeed a story about the Cosmos as the 3D sphere. I had also written a science fiction novel on this subject in 1998, entitled 'The Grand Voyage of the Potato Cosmology'. [3]

However, if the Cosmos were a 3D sphere, the telescope would turn into a microscope after a quarter of a revolution, and an inverse perspective would arise in which the more distant objects appear larger. (Fig. 11) [4] Fortunately, such a phenomenon has not yet been reported. As a geometry artist, I can say that a 3D

sphere is an elegant geometrical model, but it lacks diversity. In contrast, a 3D torus Poly-Twistor with non-positive curvature does not invert perspective and can produce a surprising diversity of forms, yet it is still elegant.

The Helical Torus worked well enough as a 'topological study assuming the macroscopic shape of the Cosmos as a 3D torus'. However, as shown in Fig. 12, an unnatural torsion was observed in the orbit near perihelion, which remained a problem to be improved.



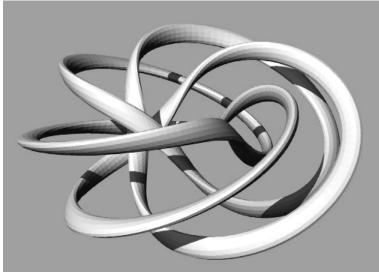


Fig. 12
Torsion of the Helical Torus perihelion

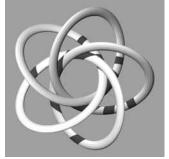
It is noticeable when the cross-section of the orbital string is a flattened ellipse. Figure shows an example at frequency 5/4.

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### 2. 1. From Villarceau's circle to Villarceau's ellipse

In general, there are three types of exact circles on the torus surface. One is the meridian. The second is the longitude. (see Figure 2). The third is known as the 'Villarceaus circle' and appears in the cross-section when sliced as shown in Fig. 13. The Villarceau's perfect circle appears when the torus minor circle (the cross-section of the arms) is a perfect circle. It is shown in the green cross section in Fig. 13. As a generalization, if the minor circle of the torus is an ellipse, as shown in the blue cross-sectional diagram in Fig. 13, it would be what we should call 'Villarceau's ellipse'. (Fig. 13)

With this Villarceau's elliptical orbit, let the entire torus rotate and undergo precession. The disturbance near the perihelion in the Helical Torus is eliminated and a smooth space curve is obtained. In distinction to the Helical Torus, this is called a 'Precessed Villarceau Torus' and is shown in Fig. 14.



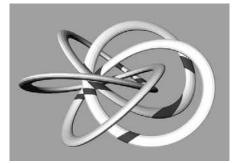


Fig. 14 Precessed Villarceau Torus as frequency 5/4

Fig. 13 Villarceau's circle and ellipse

The drawing is made by superimposing a plan view and a cross-sectional view.

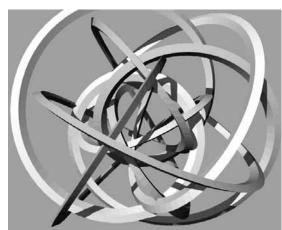
For the green torus, where the minor circle is an exact circle, the Villarceau's circle appears in the section cut by the tangent plane shown by the red line. The cross-section in the plan view is a red ellipse.

Transforming the small circle into an arbitrary ellipse and performing the same operation on the blue torus yields the Villarceau's ellipse.

All previous Poly-Twistors of the Helical Torus can be converted and refined with the Precessed Villarceau Tori. A comparison between a Tri-Twistor of the previous Helical Tori and of the Precessed Villarceau Tori is shown in Fig. 15. The unnatural twisting in the left figure has been eliminated in the right figure.



Helical Torus



Precessed Villarceau Torus

Fig. 15 Comparison of Tri-Twistor (frequency 3/2)

I could disclose a group of Poly-Twistor models converted to Precssed Villarceau Tori here, but I leave that for another paper. In this paper, let us move on with the discussion. The Precessed Villarceau Torus solved the smoothness problem, but there was one point of dissatisfaction. The center of the torus (the center of precession) does not generally coincide with the focus of the Virarceau ellipse. If we add the new assumption that "the center of precession also precesses", that problem is solved. However, there are more promising approaches, which will be discussed next.

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### 3. 1. Images of black holes

In 2021, dynamic precession of a black hole was observed. (Fig. 16) I immediately produced a model of a Helical Torus whose frequency is a convergent of the golden ratio, confirming the analogy. (Fig. 17) I intuitively realized that the curvature of space created by a singularity comparable to a supermassive black hole could make such a precessional motion possible. And it is highly possible that the myriad of visible ray or magnetic field lines are composed of only one.

The precession of the Earth orbiting a low-mass, low-density central star of about the size of the Sun has a period of tens of thousands of years, which is about the size of an error, and the orbit is generally stable and elliptical. The elliptical orbit is almost a perfect circle, which is also within the margin of error.

When the mass density of the main star reaches the level of a black hole, the precession is no longer an error, but plays a leading role.

### 3. 2. Schwarzschild precession

The precession is sometimes described as 'the dragging of an inertial system by the rotation of a central body'.

Shortly after seeing the precession of a black hole, I came across a diagram explaining the Schwarzschild precession shown in Fig. 18 on the internet and felt the need to rigorously introduce the Schwarzschild precession into Poly-Twistor. Compared to the elegance of the Schwarzschild precession, the conventional Helical Torus seems as naive as Ptolemaic cosmology.

Let us shift our perspective further to atomic structure. Sommerfeld proposed in 1913 that electrons can take elliptical orbits (Fig. 19)

Event Horizon Telescope

Fig. 16 Observed images of the vicinity of a black hole

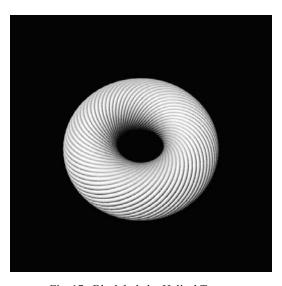


Fig. 17 Black hole by Helical Torus

In addition, the spin of the electron has been shown to have non-integer periods and to be in precessional motion.

The Poly-Twistor may be a model to unify these phenomena, and at the end of 2024 I decided to abandon both the Helical Torus and the Villarceau's Ellipse and work on recasting the Poly-Twistor on the basis of Kepler's first law, the 'elliptical orbit of a planet' and the Schwarzschild precession.

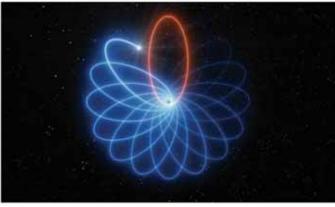


Fig. 18 Schwarzschild precession Credit: ESO/L. Calçada

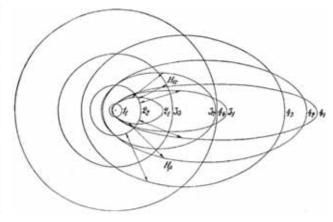


Fig. 19 Electron elliptical orbitals of the Sommerfeld From Wikipedia.

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### 4. 1. Any elliptical orbit in precessional motion

Suppose that, according to Kepler's first law, there is a mass point in elliptical motion with a singularity as its focus. Consider the precession of the entire system around the focus of that ellipse.

If the torus was allowed to precess as it was, it would be a torus as a plane figure and the trajectory would be self-intersecting. Fig. 18 was such a figure.

Therefore, as shown in Fig. 20, it is considered that the ellipse is inclined at an arbitrary angle (orbital inclination angle  $\theta$ ) to the axis of the ellipse's major diameter with respect to the precessional rotation plane, and that it is then subjected to

precessional motion around the Z-axis. Attention: the orbital inclination is usually taken to be the angle of the main star to the equatorial plane, but in this paper it is the angle between the precessional plane of rotation and the elliptical plane of orbit of the celestial body.

In this way, it does not cross itself and forms a knot, for example, as shown in Fig. 21.

A brilliant experimental device for intuitively understanding the 'gravitational field' of relativity is the metaphor of the motion of a sphere rolling on a surface of negative curvature. It is such a popular experimental device that most science museums have it permanently installed. (Fig. 22) This is a good example of how higher dimensional physical phenomena can be clarified by decreasing dimension. I never gets tired of watching this exercise. The example of Hubble's Law on the surface of a balloon is also another good metaphor.

When the trajectory of a sphere rolling on a negatively curved surface is observed, elliptical orbits as well as forward or retrograde precessional motion are commonly observed. As mentioned above, the prevailing theory is that the precessional motion is the drag of the inertial system caused by the rotation of the central body, but in this experiment there is nothing rotating at the center.

In this paper, I consider that precession in three-dimensional space is independent of the rotation of the central body, that the only thing to consider is the curvature of space due to mass and density, and that precession can occur isotropically. This is because otherwise the isotropic cage structure of electrons in atoms, which will be discussed later, cannot be explained.

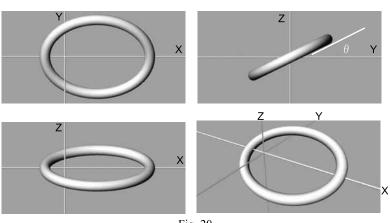


Fig. 20
The angle between the plane of precession and the elliptical orbital plane is the orbital inclination angle.



Fig. 21 Precessed elliptical orbit The cross-section of the orbit is a flattened ellipse

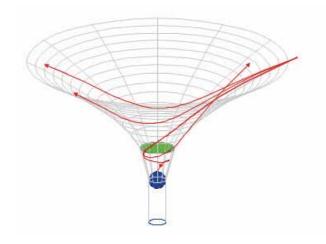


Fig. 22 Two-dimensional metaphor of the gravitational field, from the National Science Museum.

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### 4.2. Presessed Ellipse Torus

For simplicity, consider a unit ellipse with a semi-major axis of 1. (Fig. 23)

Semi-minor axis of the ellipse b (0 < b < 1)

Orbital inclination angle  $\theta$ 

A' = number of orbital revolution to complete the ring,

B' = number of precessional revolution to complete the ring,

A'/B' = orbital/precessional ratio = k



Clockwise is defined as plus direction.

The focal point of the ellipse is at a distance of  $\sqrt{1-b^2}$  on each side from the center of the ellipse.

When the orbital inclination angle  $\theta$  is minus, it is a mirror image of the plus case.

The focus of the elliptical orbit and the center of precession always coincide.

Thus a Helical Torus like shape is created and a perfectly smooth space curve is obtained. Let us call this a 'Precessed Ellipse Torus' to distinguish it from the Helical Torus and the Precessed Virarceau Torus. Compare the Precessed Ellipse Torus in Fig. 24 with the Helical Torus in Fig. 12 and the Precessed Virarceau Torus in Fig. 14. They are all topologically identical knots.

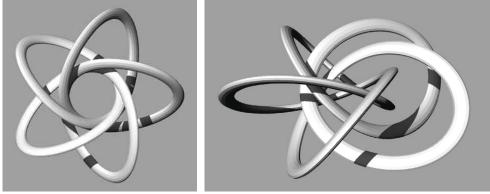


Fig.24 Precessed Ellipse Torus A'= 5, B'= - 2

Knots can be classified using Precessed Ellipse Torus, as was possible with the Helical Torus. Part of the classification table is shown in Figure 25.

m 1 igure 25.										
B'	-4	-3	-2	-1	0	1	2	3	4	5
A'			_	·	_	·		-		-
0				$\bigcirc$		$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$
1				O	0	<u></u>	<b>③</b>	9	<b>③</b>	<b>@</b>
2				0	0	<b>®</b>	<b>O</b>	9	9	6
3			0	8	0	8	9	<b>O</b>	6	6
4		$\Diamond$	0	\$	0	8	<b>®</b>	(8)	9	9
5	<b>(</b>	0	Ø	8	0	8	8	8		9

Fig. 25 Classifying knots with Precessed Ellipse Torus

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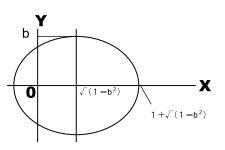


Fig. 23 Unit ellipse

It should be noted that the orbital/precessional ratio k is different from the frequency A/B in the Helical Torus.

In the Helical Torus, the knot was determined by the frequency, but when applied to the Precessed Ellipse Torus, the transformation is as follows.

$$A = A$$

$$B-A=B'$$

For example, a Helical Torus with a frequency of 5/2 corresponds to the following in the Fig 25.

$$A' = 5$$

$$B' = -3$$

When the sign + - of cos  $\theta$  and B' are different, the orbital and precessional directions are retrograde each other.

Fig. 26 reproduces the magnetic field line pattern of the black hole in Fig. 16 with an Precessed Ellipse Torus. Differences can be observed in the orbit near the center from the model in Fig. 17. The parameter values are as follows.

$$A' = 34$$

$$B' = -5$$

$$b = 0.866$$

$$\theta = \pi/3$$

Surprisingly, it was of a type where the orbital and precessional directions were retrograde. It is, of course, a Precessed Elliptse Torus composed of a single link.

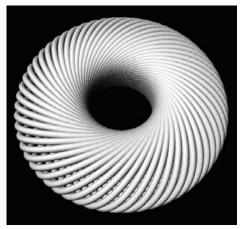


Fig. 26

As a sideshow, I also recreated a galactic spiral (Fig. 27). This structure has no flip side. The vortex shape does not change even if it is turned upside down 180 degrees. I have seen many photographs of galaxies, and the overwhelming majority of them are of type A' = 2.





Fig.27 A' = 2, B' = 55, b = 0.9,  $\theta = \pi/9$  Precesed Ellipse Torus

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### 4. 3. Poly-Twistor using the Precessed Ellipse Torus

Now it is time to reconstruct the Poly-Twistor using the Precessed Ellipse Torus. The operations from this point onwards are no different from those of the initial Helical Torus version. Based on the symmetry of Plato's regular polyhedron, all we have to do is rotate it around the x-, y- and z-axes at the appropriate angles. The chirality + - distinction between any two Precessed Ellipse Torus is exactly the same as Fig. 8.

Some examples of Poly-Twistor reconstructed in Precessed Ellipse Torus are shown below, representing each of the above eight types: Tri, Tetra+, Tetra-, Hexa+, Hexa-, Deca+, Deca-, and XV. (Fig. 28)

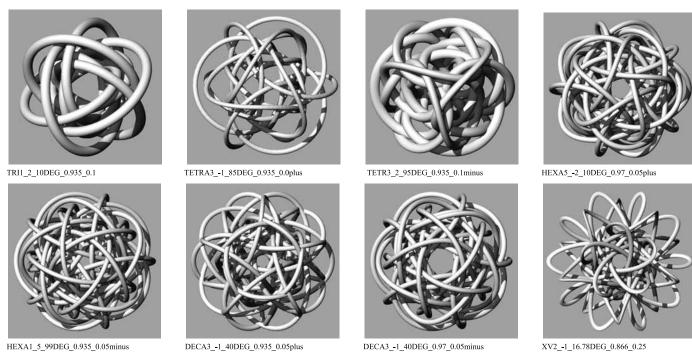


Fig. 28 Poly-Twistor of the Precessed Ellipse Torus

After various trials, it is more interesting when the orbital direction and precessional direction are retrograde each other. This is probably because there are more gaps, due to the smaller number of windings. The work shown in Fig. 1 at the beginning of this paper is such an example. Fig. 29 shows the actual 3D printer model of it, with the parameters as follows.

Hexa-Twistor + of the Precessed Ellipse Torus A' = 5 B' = -3 b = 0.935  $\theta$  = 2/9\*  $\pi$ 

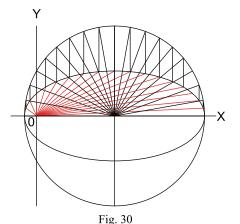
Countless more will be produced in the future.

### 5. 1. Kepler's second law: Area velocity constant

In the Precessed Ellipse Torus, the elliptic motion is plotted for simplicity in the manner shown in Fig. 30. The method is to set up a reference circle corresponding to the given ellipse, make an uniform angular velocity motion in the reference circle and map it onto the ellipse. Of course, this is not an area-velocity constant division.



Fig.29 A real model of Hexa Twistor+ of the Precessed Ellipse Torus



Elliptical orbit drawing area velocity is not constant.

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However, in nature, from celestial bodies to electrons, they follow elliptical orbits in accordance with Kepler's second law of constant area velocity. The further away from the focal point, the slower it travels, and the closer it is to the focal point, the greater its velocity, so the shape of the space curve should naturally change when it is subjected to precessional motion.

### 5. 2. Poly-Twistor using the Precessed Kepler Torus

I have therefore strictly reproduced an elliptical orbit based on the law of constant area velocity [5] and made it move precessionally. We shall call this the 'Precessed Kepler Torus'. Then lets re-construct a Poly-Twistor of the Precessed Kepler Tori. Fig. 31 and 32 show the development process. To make the law of constant area velocity easier to understand, the cross section of the orbital string was made equilateral triangles, and the orbits at difference times were displayed discretely.

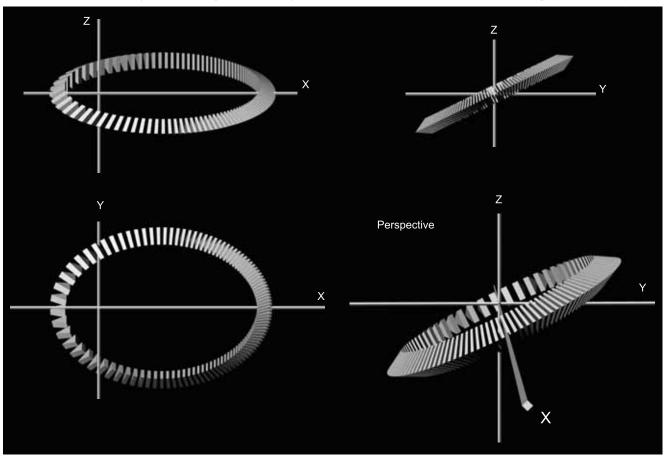


Fig.31 Elliptical orbit based on the law of constant area velocity

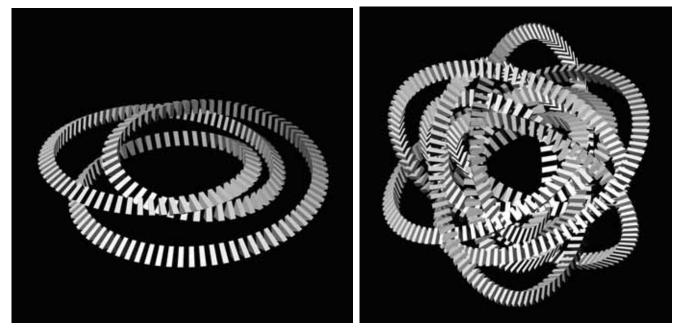
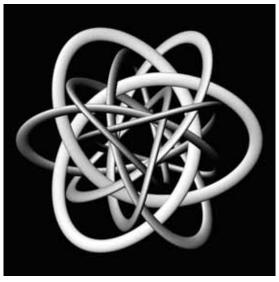


Fig. 32 Precessed Kepler Torus and Tri-Twistor constructed with them. (A' = 2, B' = 1, b = 0.935,  $\theta = \pi/18$ )

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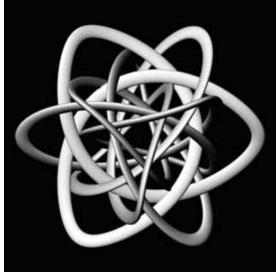


Fig. 33 Output and comparison of Tetra-Twistor with Precessed Ellipse Torus (left) and Precessed Kepler Torus (right)

Let us output and compare the Tetra-Twistor with the same parameters in Precessed Ellipse Torus and Precessed Kepler Torus. How do the readers feel when looking at Fig. 33? I felt that the Precessed Kepler Torus was unnatural. The farther from the focal point, the greater the influence of the precession can be seen. This is due to the simplicity of the model setting. In nature, the effect of precession should weaken as one moves away from the central object (according to the theory of relativity, it is inversely proportional to the fourth power of the distance), but this model does not take this into account. In addition, the longer the orbital radius, the more slowly the object moves, so it is affected by the precession for a longer period of time. In addition, the positional shift due to precession increases in proportion to the orbital radius, and the vicinity of the aphelion is doubly and triply affected by excessive precession. This means that the precession is deformed.

Therefore, it may not have been necessary to introduce a law of constant area velocity in the present study. The Precessed Ellipse Torus may be more in line with the actual orbit. This is because the error in ignoring the area velocity and the error in ignoring the influence gradient of the precession, which is inversely proportional to the fourth power of the orbital radius, cancel each other out.

Nevertheless, the Precessed Kepler Torus using the law of constant area-velocity created Fig. 34 by chance. It shows a supple curve as if the G clef were in precession. It can be said to be an unintentional deformation effect.

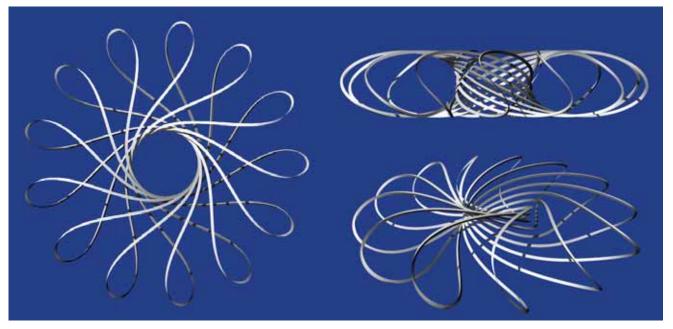


Fig.34 A Precessed Kepler Torus by chance

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### 6. 1. Conclusions

Four techniques of the Poly-Twistor series have been produced in this study.

Helical Torus

Precessed Villarceau Torus

Precessed Ellipse Torus

Precessed Kepler Torus

Countless more models will be found in the future using these methods.

The Helical Torus version of the Poly-Twistor, inspired by the discoveries of Plato, Riemann, Lobachevsky, Poincaré, Einstein, Hubble and Penrose, began with the question of what possibilities there are for topological forms in a finite, boundary-less universe and developed into a classification of three-dimensional tori. Naturally, the focus shifted to black holes, galaxies, precession of planets and electron orbits of atoms, and the present paper introduced Precessed Ellipse Torus to incorporate the discoveries of Kepler, Newton and Schwarzschild. I feel that the 'law of gravity', which works universally from the macroscopic to the microscopic aspects of the Cosmos, has been enclosed in a 3D torus. I have presented it as 'A Form of the Cosmos, A Form of the Atom', but it might be better to call it 'A Form of the Gravitational Field' in a more general way.

The following is a list of the results of this research and emerging projections.

### 6. 2. Interpretation as electron orbitals

Poly-Twistor explains how multiple electron orbitals do not cross each other and fold together in the atomic structure. It can be considered a more generalized scheme of de Broglie's standing wave. Let's call this the 'Precessed Ellipse Torus Standing Wave Hypothesis'. Natural elements, with atomic numbers ranging to 94, would not be able to balance multiple electrons and nucleons without the use of regular polyhedral symmetry.

### 6. 3. The Precessed Ellipse Torus is the best solution

Of the Helical Torus, Precessed Villarceau Torus, Precessed Ellipse Torus and Precessed Kepler Torus, the Precessed Ellipse Torsu is the most promising model. The precession center and the focus of the elliptic orbit always coincide. The Precessed Kepler Torus, which introduced the law of constant area velocity, resulted in a deformation of the precession effect and was counterproductive.

### 6. 4. Hypothesis that precession center also precesses

The Precessed Villarceau Torus generally do not coincide with the Precession center and focus of the elliptical orbit. That is one area where it is inferior to the Precessed Ellipse Torus. However, a hypothesis that is hard to discard is that when multiple Precessed Ellipse Torus entangle and influence each other in a Poly-Twistor-like fashion, the center of the precession may itself precessionally move, resulting in a Precessed Villarceau Torus. This is a new idea emerged by the present study. [6]

### 6. 5. Definition of orbital inclination angle

The definition of the angle between the precession plane and the orbital plane as the inclined orbital angle is unique.

The case where the precession direction and orbital direction are opposite is not exceptional. The observation images of black holes actually show this type of case. The vortexes of magnetic field lines, which appear to be innumerable, may be traced in a single link, like the Precessed Ellipse Torus.

General magnetic field lines may also form a single Precessed Ellipse Torus. The orbital inclination angle may generate polarity and rotational forces such as motors.

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### 6. 6. Orbit strings

When the trajectory of a Precessed Ellipse Torus is represented by a thick 'string', its shape of cross section are secondary parameters. If the cross section of the string is a flat ellipse instead of a perfect circle, a beautifully smooth curved surface is obtained. If the cross-section is polygonal, it will be a malleable surface and could be produced by paper. In fact, I has previously produced a Hexa-Twistor composed of the Helical Tori with an equilateral triangular cross-section by paper. (Fig. 35) [7].

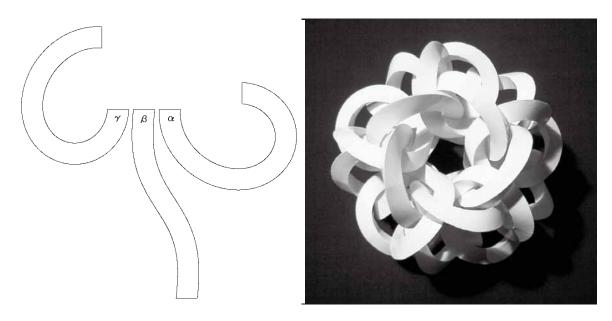


Fig. 35 Handmade of paper Hexa-Twistor Triangular Section (2003)

### 7. 1. Appendix

Poly-Twistor's sculptures are inspired by classical physics, relativity, quantum theory, cosmology, etc., and are abstract in nature, exploring geometrically unknown structures. It can be regarded as pure abstract sculpture. It is not intended as a rigorous demonstration of real physical phenomena.

However, geometrical and abstract realities that are thoroughly devoid of arbitrariness will sooner or later be discovered in nature. I have had such an experience many times.

Poly-Twistor research began in 1997, has continued for almost a quarter of a century and has been published successively, I have yet to see any reports of other researcher or artist reproducing it. This is not because people is not interest in this research. Rather, Poly-Twistor is an unusually popular piece of my work, and when I exhibit it, it can sell out in an instant, and has been stolen several times. Despite such interest, it would be difficult to do replication study. The ability of professional scientists to process mathematical formulae is outstanding, but I have often felt that their imagination and intuition are, in comparison, too naive in my view. What is self-evident in 2D and 3D modeling, they sometimes seem to be complicating the problem unintentionally, endlessly playing with formulas for this, that and the other.

In the conventional scientific community, conclusions tend to be rejected in the absence of anyone to follow up on them, but in geometrical studies such as mine, the computer, more than anything else, has objectively proven its reproducibility. A syllogistic proof is not always essential. The geometric output, which is free from arbitrariness, makes the truth of the hypothesis obvious.

As someone who has been working with these geometric forms for many years, I may be allowed to speak out. I will try to describe it unabashedly below. I would be delighted if I could create a stir in the scientific community.

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### 7. 2. Atomic Form Universal Singularities

While continuing the 'geometric thinking' represented by Poly-Twistor on a daily basis, and being exposed to scientific findings that are updated every year, a new image of the atom that integrates these findings naturally emerges.

If a nucleus composed of protons and neutrons were the size of a sugar cube, it would have a mass of up to 250 million tons. If the whole earth is the size of a single atom, the nucleus is equivalent to a sphere about 200 meters in diameter. If the sphere were packed with neutrons without gaps, its mass would be dense enough to compress the entire mass of the Earth. It is like a small neutron star. In that case, it is calculated that a black hole with a Schwarzschild radius of about 8 millimeters could exist at the center of a sphere 200 meters in diameter. Neutron stars can be metaphorically described as 'giant nuclei', but the reverse is also true: nuclei are microscopic neutron stars.

If the nucleus is a sphere with a diameter of 200 meters, the electrons correspond to a size of about 10 centimeters in diameter and form a basket (Poly-Twistor) woven with standing waves in elliptical orbits with aphelion around the stratosphere. The space of about 5000 kilometers between the nucleus and the electron orbital layer is almost a 'perfect vacuum'. This means that most of the matter we come into contact with on a daily basis is almost scanty and empty. This is why neutrinos can pass through even the earth. Despite this, we are not invisible man because the cage of electron orbitals makes the atoms so bulky that they act as an electromagnetic barrier, causing steric hindrance between atoms.

The absolute mass of the nucleus is tiny, but its density is comparable to that of a neutron star or black hole. The vicinity of the nucleus is curved so violently with negative curvature that Kepler's law and relativistic precession play a leading role, and the electron orbits may have dynamic and elegant precession elliptical orbits.

Even a nucleus as dense as a neutron star, when encased in a cage of electron orbits and bulky, can fly through the air like dust and even drift through space. So at microscopic sizes gravity is usually considered negligibly weak compared to the other three forces (electromagnetic interaction, weak interaction and strong interaction). However, it is my thinking that this would be very misguided.

The nucleus is an unmistakable singularity with a density comparable to that of a neutron star. You could call it a tiny black hole. In other words, we are surrounded by countless omnipresent tiny black holes.

If we take this idea forward, the center of every celestial body is also a singularity leading to an 'almost infinite' density. And the average density of the universe is as dilute as one proton per cubic meter - in short, the universe is almost as empty as an atom.

In this study, we were able to formatively relive that the action of 'gravity', as represented by Kepler's laws and precession, is the prevailing platform in the entire universe, from the macroscopic to the microscopic.

We look forward to the day when a true unified theory will eventually be completed in physics.

### Note

### [1]

Previously published papers about the Poly-Twistor are as follows.

Akio Hizume, Hexa-Twistor, MANIFOLD #01, pp. 10-12, 2000. (in Japanese)

Akio Hizume, Poly-Twistor, MANIFOLD #04, pp. 8-9, 2002. (in Japanese)

Akio Hizume, Poly-Twistor, ISAMA Proceedings, 2002. (This paper was reprinted on "inter-native architecture OF music", Star Cage Publishing, pp. 199-204, 2006.)

Akio Hizume, Hexa-Twistor Trianglar Section, MANIFOLD #05, 2002.

Akio Hizume, Real Model of the POLY-TWISTOR Triangular-Section, MANIFOLD #06, 2003.

Akio Hizume, Hexa-Twistor Trianglar Section, 6th Bridges Proceedings, 2003.

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Akio Hizume, Yoshikazu Yamagishi and Shoji Yotsutani, Poly-Twistor for Rapidprototyping, MANIFOLD #22, pp. 5-10, 2012. (in Japanese)

Akio Hizume, Yoshikazu Yamagishi and Shoji Yotsutani, Poly-Twistor by 3D Printer Classification of 3D Tori, Proceedings of Bridges, pp. 555-558, 2013.(\* The reason why the papers from this period are jointly named is that Ryukoku University provided support for travel expenses to international conferences, etc.. The content of the papers is 100 per cent my own work).

### [2]

500 copies of the catalogue Poly-Twistor Periodic Table were printed and 300 copies distributed as an Exchange Gift with the presentation at the 2018 biennial Gathering for Martin Gardner in Atlanta, USA. A PDF version is available online. It is also available online.

http://starcage.org/poly-twistors\_s.pdf

### [3]

In the novel The Grand Voyage of the Potato Cosmology (1998), the theory was developed based solely on Hubble's Law. According to the established theory at the time, the expansion of the universe reached the speed of light at a point 15 billion light years away and the Big Bang occurred 15 billion years ago. The novel made the Big Bang unnecessary and instead calculated the 'half-life of the Universe'. This means that 10.4 billion years ago everything was uniformly half the size of everything else, an original hypothesis that can be traced back to an infinite past. Conversely, after 10.4 billion years, everything will expand to twice its size. No one will notice because ruler will also expand. This hypothesis is becoming increasingly plausible. At the time of writing this novel, I naively expected the Cosmos to be a 3D sphere. As noted in this paper, I now expect the global structure of the universe to be one of a 3D torus. The PDF of the novel is available below.

http://www.starcage.org/jagaimo.pdf



(in Japanese)

This possibility is suggested in George Gamow's Tomkins in Wonderland, translated by Koji Fushimi (1950, Hakuyosha). Reading this, I began to question the 3D spherical theory of the Cosmos.

### [5]

The method for calculating the time variation of elliptical orbits based on the law of constant area velocity was taken from the following website.

Hirosaki University website, Understanding relativity and its surroundings.

https://home.hirosaki-u.ac.jp/relativity/1881/

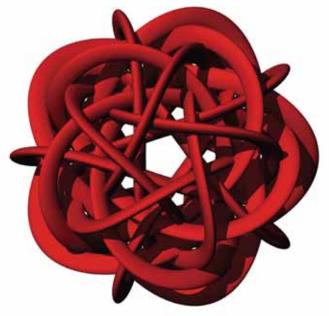
### [6]

A body orbiting a main star may, from a local perspective, intend to continue its elliptical orbit faithfully and forever, but in the big picture, the inertial system itself is rotating, and that system rotates in a larger gravitational field, and so on..

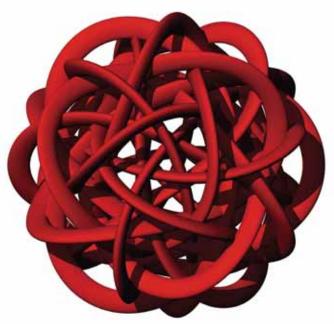
There is no objectively observable fixed point anywhere in the universe. This is one kind of 'dynamic equilibrium'. The possibility of the central axis of precession itself precessing may also be an example of a new 'dynamic equilibrium'. It recalls the technique of squeezing many balloons into one box at once to calm them down.

### [7]

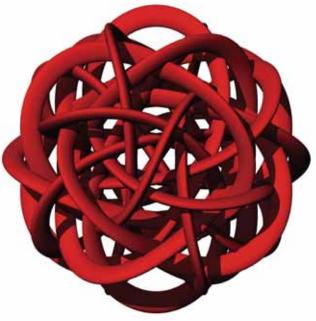
Before 3D printers, it took three months of intensive work to complete this paper model. It was a last-ditch operation, where even the dampness of the fingers could buckle the paper. It is this production that nobody will ever be able to follow up. I myself don't want to make it again. We live in a better time with the widespread use of 3D printers.



5-fold of HEXA23\_-2\_5\_0.6\_0.6\_500\_0.1\_0.05plusVillarceau



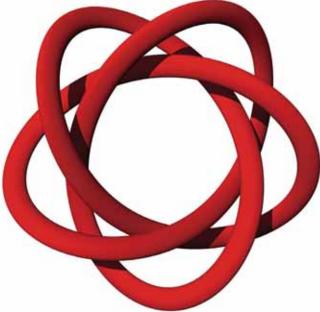
3-fold of HEXA23\_-2\_5\_0.6\_0.6\_500\_0.1\_0.05plusVillarceau



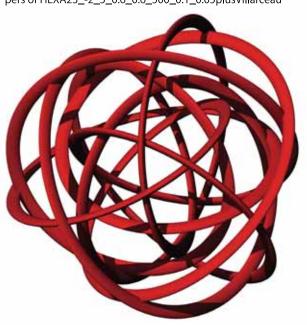
2-fold of HEXA23\_-2\_5\_0.6\_0.6\_500\_0.1\_0.05plusVillarceau



pers of HEXA23\_-2\_5\_0.6\_0.6\_500\_0.1\_0.05plusVillarceau



23\_-2\_5\_0.6\_0.6\_500\_0.1\_0.05plus Precessed Villarceau Torus



3-fold of TRI23\_1\_2\_1\_0.6\_500\_0.06\_0.03Villarceau

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